

2 Algebra and graphs (continued)

2.11 Sketching curves

Recognise, sketch and interpret graphs of the following functions:

- (a) linear
- (b) quadratic
- (c) cubic
- (d) reciprocal
- (e) exponential.

Notes and examples

Functions will be equivalent to:

- $ax + by = c$
- $y = ax^2 + bx + c$
- $y = ax^3 + b$
- $y = ax^3 + bx^2 + cx$
- $y = \frac{a}{x} + b$
- $y = ar^x + b$

where a , b and c are rational numbers and r is a rational, positive number.

Knowledge of turning points, roots and symmetry is required.

Knowledge of vertical and horizontal asymptotes is required.

Finding turning points of quadratics by completing the square is required.

x-intercepts (y=0)

2.12 Functions

- 1 Understand functions, domain and range, and use function notation.

- 2 Understand and find inverse functions $f^{-1}(x)$.

- 3 Form composite functions as defined by $gf(x) = g(f(x))$.

Notes and examples

Examples include:

- $f(x) = 3x - 5$
- $g(x) = \frac{3(x+4)}{5}$
- $h(x) = 2x^2 + 3$.

e.g. $f(x) = \frac{3}{x+2}$ and $g(x) = (3x+5)^2$. Find $fg(x)$.

Give your answer as a fraction in its simplest form.

Candidates are **not** expected to find the domains and ranges of composite functions.

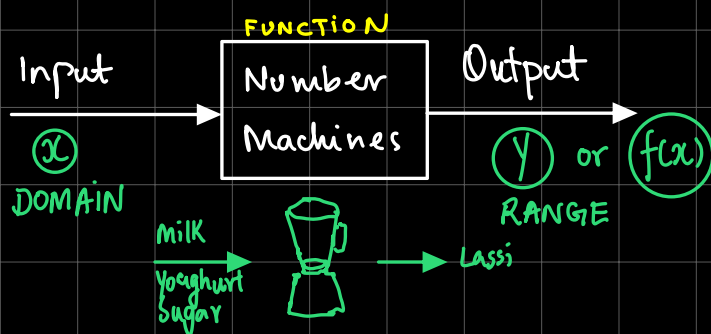
This topic may include mapping diagrams.

FUNCTIONS

(4 MARKS) (P1)

In old syllabus Functions was different for OLEVELS (4024) and IGCSE (0580).

From June 2025 onwards its same for both.



$$f(x) = 2x + 5$$

↓ Name of function
↓ value of input (x)

$$g(x) = x^2 + 3$$

$$1) f(3) = 2(3) + 5 = 11$$

$$5) g(t) = t^2 + 3$$

$$2) g(2) = (2)^2 + 3 = 7$$

$$6) f(k) = 2k + 5$$

$$3) f(10) = 2(10) + 5 = 25$$

$$7) f(t-3) = 2(t-3) + 5$$

$$4) g(5) = (5)^2 + 3 = 28$$

$$8) g(k+1) = (k+1)^2 + 3$$

OLD SYLLABUS $\rightarrow$ $f(x)$ = (This is only symbols we will use)
 $\rightarrow$ $f:x \rightarrow$ (This will never be used now)

TYPE 1

Q:

$$f(x) = 2x + 5$$

Given that $f(3) = t + 2$, find t .

$$f(3) = t + 2$$

$$2(3) + 5 = t + 2$$

$$11 = t + 2$$

$$9 = t$$

$$\boxed{t = 9}$$

Q:

$$g(x) = 3x - 5$$

Given that $g(k) = 12$, find k .

$$g(k) = 12$$

$$3k - 5 = 12$$

$$3k = 17$$

$$k = \frac{17}{3}$$

Q:

$$f(x) = 4x - 3$$

Given that $f(t) = t$, find t .

$$f(t) = t$$

$$4t - 3 = t$$

$$4t - t = 3$$

$$3t = 3$$

$$t = 1$$

Q:

$$g(x) = 2x - 3$$

Given that $g(k) = k$, find k .

$$g(k) = k$$

$$2k - 3 = k$$

$$2k - k = 3$$

$$k = 3$$

Q.

$$f(x) = 3x + 7$$

Given that $f(3t+1) = 12t$ find t .

$$f(3t+1) = 12t$$

$$\underline{3(3t+1)} + 7 = 12t$$

$$9t + 3 + 7 = 12t$$

$$10 = 12t - 9t$$

$$10 = 3t$$

$\frac{10}{3}$	$= t$
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TYPE 2

INVERSE OF A FUNCTION

Symbol: $f^{-1}(x)$

Beware: f^{-1} means
put $x = -1$ in function
 $f(x)$

STEPS:

1) Replace $f(x) \rightarrow y$

2) Make x subject

3) Replace $x \rightarrow f^{-1}(x)$
 $y \rightarrow x$

Q: $f(x) = 2x - 3$
Find inverse.

$$f(x) = 2x - 3$$

$$y = 2x - 3$$

$$y + 3 = 2x$$

$$x = \frac{y + 3}{2}$$

$$f^{-1}(x) = \frac{x + 3}{2}$$

Q:

$f(x) = 12 - 2x$, Find inverse.

$$y = 12 - 2x$$

$$y + 2x = 12$$

$$2x = 12 - y$$

$$x = \frac{12 - y}{2}$$

$$f^{-1}(x) = \frac{12 - x}{2}$$

Q.

$$f(x) = \frac{2x+3}{5x}$$

Find inverse.

$$y = \frac{2x+3}{5x}$$


$$5xy = 2x + 3$$

Bring all x terms to one side

$$5xy - 2x = 3$$

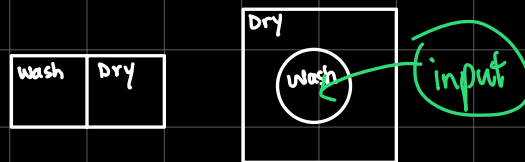
$$x(5y - 2) = 3$$

$$x = \frac{3}{5y-2}$$

$$f^{-1}(x) = \frac{3}{5x-2}$$

Type 3 Composite Functions.

(Function within a function)



Q: $f(x) = 2x + 3$

$g(x) = 5x^2 + 6$

Step 1: Write first function with () instead of x .

Step 2: Put the value of second function in ().

(i) $f(g(x)) = 2(5x^2 + 6) + 3$
 $= 10x^2 + 12 + 3$
 $= 10x^2 + 15$

First $\swarrow$
Second $\downarrow$

(ii) $g(f(x)) = 5(2x + 3)^2 + 6$
 $= 5[(2x)^2 + 2(2x)(3) + (3)^2] + 6$
 $= 5[4x^2 + 12x + 9] + 6$
 $= 20x^2 + 60x + 45 + 6$
 $= 20x^2 + 60x + 51$

Just for practice of first step. Don't simplify.

Q: $f(x) = 2x + 1$ $g(x) = (3x + 5)^2$

(i) $fg(x) = 2((3x + 5)^2) + 1$

(ii) $gf(x) = (3(2x + 1) + 5)^2$

(iii) $ff(x) = 2(2x + 1) + 1$

(iv) $gg(x) = (3((3x + 5)^2) + 5)^2$

No need to simplify.

19 $f(x) = 5x + 2$ $g(x) = x^2 - 5$ $h(x) = 7 - x$.

(a) Find $f(3)$. $= 5(3) + 2 = 17$

..... 17 [1]

(b) Find $gf(x)$.

Give your answer in the form $ax^2 + bx + c$.

$$\begin{aligned} gf(x) &= (5x + 2)^2 - 5 \\ &= (5x)^2 + 2(5x)(2) + 2^2 - 5 \\ &= 25x^2 + 20x + 4 - 5 \\ gf(x) &= 25x^2 + 20x - 1 \\ &\quad ax^2 + bx + c \end{aligned}$$

..... [3]

$f(x) = 5x + 2$ $g(x) = x^2 - 5$ $h(x) = 7 - x$.

(c) Solve $\frac{3}{hf(x)} = -1$.

$$\begin{aligned} hf(x) &= 7 - (5x + 2) \\ &= 7 - 5x - 2 \end{aligned}$$

$$\frac{3}{5-5x} = -1$$

$$hf(x) = 5 - 5x$$

$$3 = -1(5 - 5x)$$

$$3 = -5 + 5x$$

$$8 = 5x$$

$$x = \frac{8}{5}$$

$x =$ [3]

GRAPHS OF FUNCTIONS

1 LINEAR

Vertical
 $x = \square$

$y = \square$
Horizontal

gradient is +ve

① $y = -3x + 2$

② $y = 2x + 3$ ✓

gradient is positive

gradient

$y = mx + c$

SLANTING

gradient is -ve

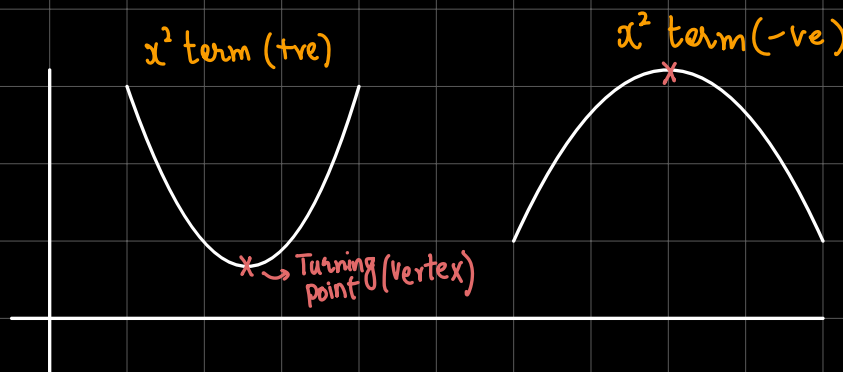
HOW TO SKETCH A SLANT LINE:
Make a table and take 3 values of x

x	0	1	2
y			

2 QUADRATIC

$y = ax^2 + bx + c$ → $y = a(x+b)^2 + c$

(STANDARD FORM) (COMPLETED SQUARE FORM)



$y = 2x^2 - 9x - 7$

$y = 3 - x^2$

Review: COMPLETED SQUARE FORM

STANDARD $\longrightarrow$ COMPLETED SQUARE

Q: Express $2x^2 - 12x + 20$ in form $a(x-b)^2 + c$.

$$\begin{aligned}
 & 2 \left[x^2 - 6x + (3)^2 - (3)^2 + 10 \right] \\
 & \quad \quad \quad \text{Half} \\
 & \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 & 2 \left[(x-3)^2 - 9 + 10 \right] \\
 & \quad \quad \quad \uparrow \quad \quad \uparrow \\
 & 2 \left[(x-3)^2 + 1 \right] \\
 & 2(x-3)^2 + 2 \\
 & a(x-b)^2 + c
 \end{aligned}$$

STANDARD FORM

COMPLETED SQUARE

Q: Express $3x^2 - 24x + 18$ in form $a(x+b)^2 + c$.

$$\begin{aligned}
 & 3 \left[x^2 - 8x + (4)^2 - (4)^2 + 6 \right] \\
 & \quad \quad \quad \text{Half} \\
 & \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \\
 & 3 \left[(x-4)^2 - 16 + 6 \right] \\
 & \quad \quad \quad \uparrow \quad \quad \uparrow \\
 & 3 \left[(x-4)^2 - 10 \right] \\
 & 3(x-4)^2 - 30 \\
 & a(x+b)^2 + c
 \end{aligned}$$

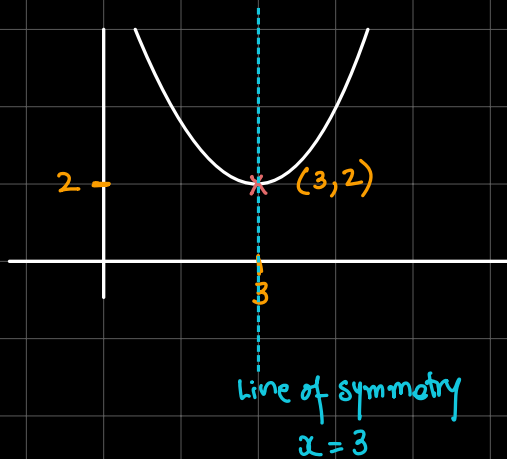
HOW TO SKETCH A QUADRATIC CURVE

$$y = ax^2 + bx + c$$

$$y = 2x^2 - 12x + 20$$



1) Shape:



$$y = a(x+b)^2 + c$$

$$y = 2(x-3)^2 + 2$$

$$x-3=0$$

Turning point =

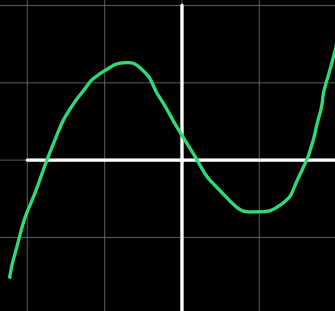
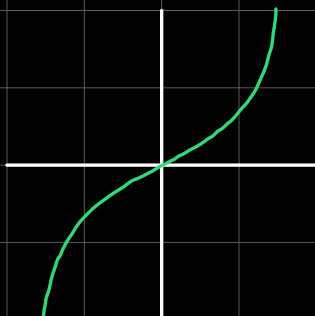
$$x=3$$

$$y=2$$

3 CUBIC GRAPHS

$$y = x^3$$

$$y = 3x^3 + 2x^2 + 2x + 1$$

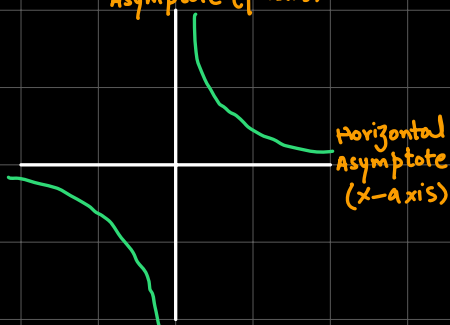


Asymptote is a line which a graph keeps getting closer and closer to but can never ever touch.

4 RECIPROCAL GRAPHS

$$y = \frac{1}{x}$$

Vertical Asymptote (y-axis)

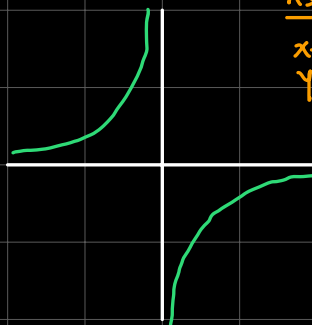


Horizontal Asymptote (x-axis)

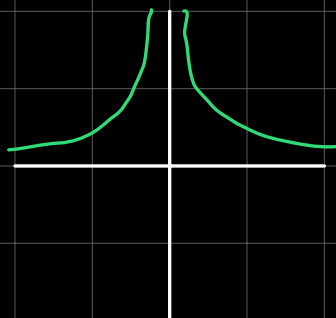
$$y = -\frac{1}{x}$$

Asymptotes

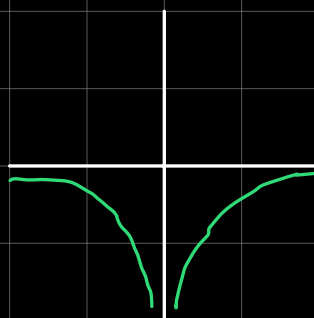
x-axis
y-axis



$$y = \frac{1}{x^2}$$



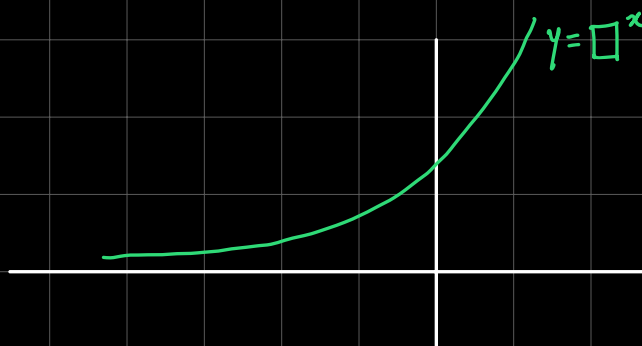
$$y = -\frac{1}{x^2}$$



5 EXPONENTIAL GRAPHS

$$y = \square^x$$

eg $y = 2^x$, $y = 3^x$



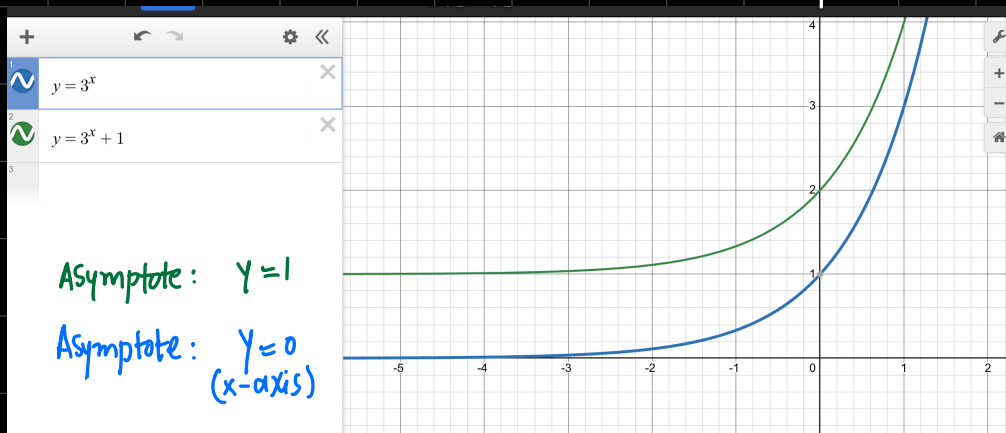
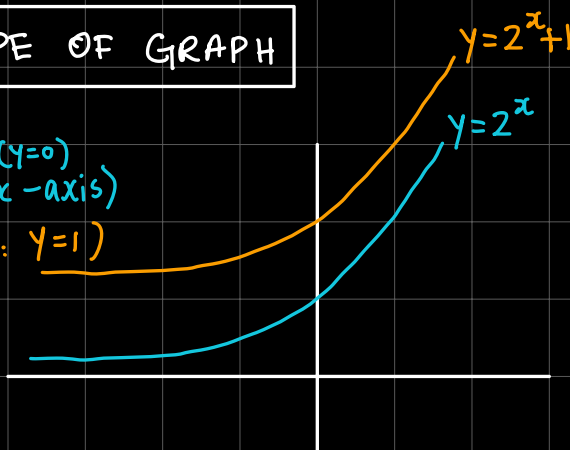
asymptote: x -axis.

KEY CHANGES TO SHAPE OF GRAPH

EX1:

$$y = 2^x \quad (\text{Asymptote: } x\text{-axis})$$

$$y = 2^x + 1 \quad (\text{Asymptote: } y = 1)$$



HOW TO IDENTIFY CORRECT GRAPH

- 1) SHAPE
- 2) y -intercept (point where graph touches y -axis) (Put $x=0$)
- 3) x -intercept (point where graph touches x -axis) (Put $y=0$)
(ROOTS)

1

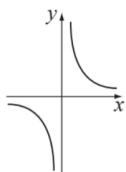


Figure 1

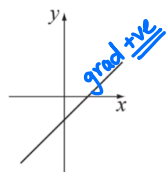


Figure 2

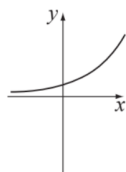


Figure 3

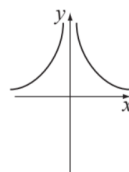


Figure 4

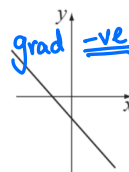


Figure 5

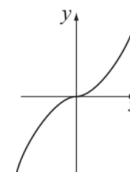
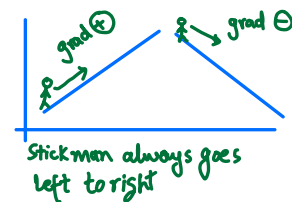


Figure 6

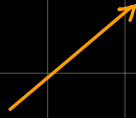
Which of the graphs shown above could be the graph of

- (a) $y = x^3$, Figure 6
- (b) $y = \frac{1}{x^2}$, Figure 4
- (c) $y = x - 1$? (slant line) (Figure 2)
grad = +1



$$y = 1x - 1$$

gradient = 1
(+ve)



3

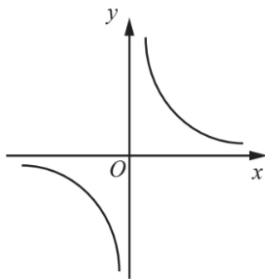


Figure A

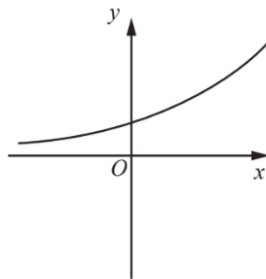


Figure B

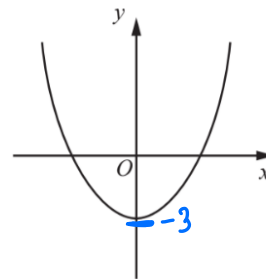


Figure C

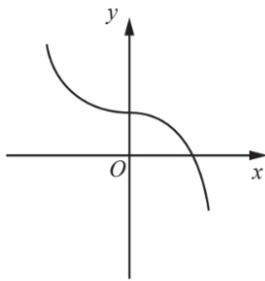


Figure D

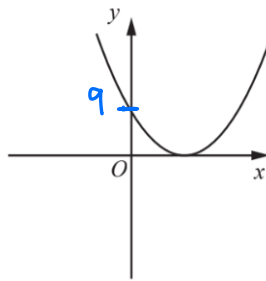


Figure E

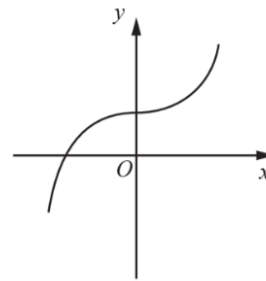


Figure F

State which figure could be the graph of

(a) $y = x^3 + 1$,

Answer Figure **F** [1]

(b) $y = x^2 - 3$, put $x = 0$
 $y = 0^2 - 3 = -3$

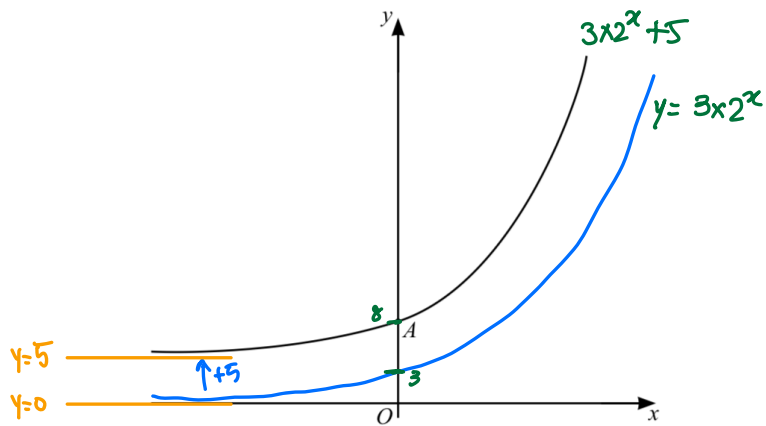
Answer Figure **C** [1]

(c) $y = 3^x$,

Answer Figure **B** [1]

(d) $y = (x - 3)^2$, put $x = 0$
 $y = (0 - 3)^2 = (-3)^2 = 9$

Answer Figure **E** [1]



The diagram shows a sketch of the graph of $y = 3 \times 2^x + 5$.

- (a) The graph crosses the y-axis at point A. (y-intercept) ($x=0$)

Find the coordinates of point A.

$$y = 3 \times 2^0 + 5$$

$$y = 3 \times 1 + 5$$

$$y = 3 + 5 = 8$$

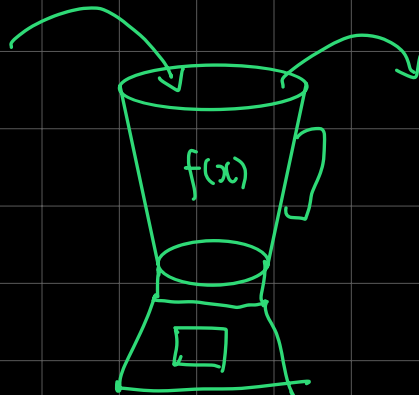
(.....,) [2]

- (b) Write down the equation of the asymptote to the graph of $y = 3 \times 2^x + 5$.

..... $y = 5$ [1]

Domain:

Milk
Sugar
Chocolate
Ice cream



Range:

Chocolate
Milk shake.

(inputs) (outputs)
DOMAIN AND RANGE

DOMAIN VALUES OF x FOR WHICH A FUNCTION
(allowed inputs) DOES NOT GO CRAZY.

TYPE 1

$$f(x) = \sqrt{x}$$

Domain: $x \geq 0$

$$f(x) = \sqrt{x-2}$$

$$x-2 \geq 0$$

$$x \geq 2$$

Domain $x \geq 2$

**GENERAL
RULE**

$$f(x) = \sqrt{\quad}$$

Domain: $\square \geq 0$
Solve this

$$1) f(x) = \sqrt{2x+3}$$

Domain: $2x+3 \geq 0$

$$2x \geq -3$$

$$x \geq -\frac{3}{2}$$

$$2) f(x) = \sqrt{2x-4}$$

Domain: $2x-4 \geq 0$

$$2x \geq 4$$

$$x \geq \frac{4}{2}$$

$$x \geq 2$$

TYPE 2

$$f(x) = \frac{3}{x}$$

$$\text{Domain: } x \neq 0$$

$$f(x) = \frac{3}{x-2}$$

$$x-2 \neq 0$$

$$x \neq 2$$

$$\text{Domain: } x \neq 2$$

GENERAL RULE.

$$f(x) = \frac{\text{Anything}}{\boxed{}}$$

$$\text{Domain: } \boxed{} \neq 0$$

Solve

1) $f(x) = \frac{2x+5}{3x-1}$

$$\text{Domain: } 3x-1 \neq 0$$

$$3x \neq 1$$

$$x \neq \frac{1}{3}$$

2)

2) $f(x) = \frac{4x}{3x-5}$

$$\text{Domain: } 3x-5 \neq 0$$

$$3x \neq 5$$

$$x \neq \frac{5}{3}$$

RANGE: VALUES OF y THAT GRAPH OF $f(x)$ COVERS.

GOLDEN RULE: NEVER TELL RANGE OF A FUNCTION WITHOUT LOOKING AT ITS GRAPH.

Just like we can not tell the flavour of milkshake (output $\rightarrow$ range) by looking at blender (function).

4

$$f(x) = 3x - 5$$

The domain of $f(x)$ is $\{-3, 0, 2\}$.

Find the range of $f(x)$.

input: $x = -3$, $f(-3) = 3(-3) - 5 = -14$ (output)

input: $x = 0$, $f(0) = 3(0) - 5 = -5$ (output)

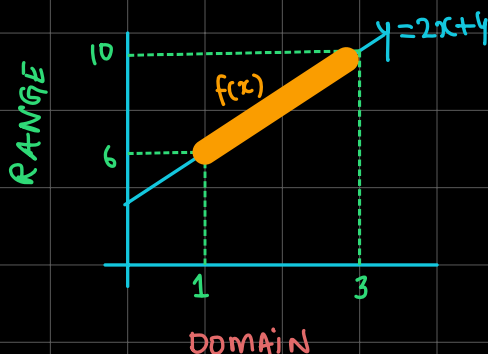
input: $x = 2$, $f(2) = 3(2) - 5 = 1$ (output)

$$\{-14, -5, 1\} [2]$$

Q: $f(x) = 2x + 4$

$$1 < x < 3$$

Find range of $f(x)$



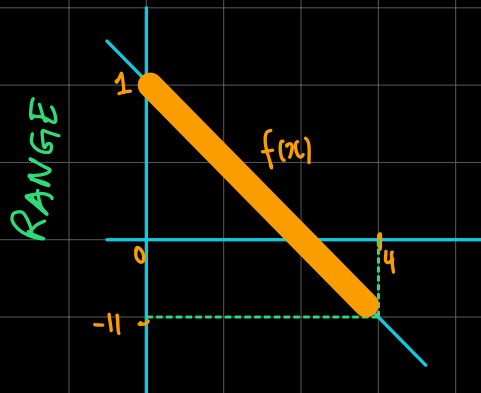
$$x = 1, y = 2(1) + 4 = 6$$

$$x = 3, y = 2(3) + 4 = 10$$

$$\text{Range: } 6 < f(x) < 10$$

Q: $f(x) = -3x + 1$ $0 < x < 4$

Find range of $f(x)$



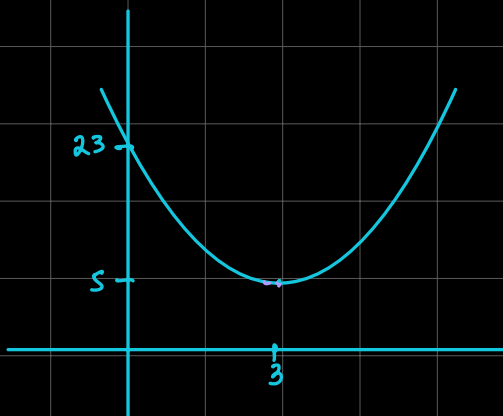
input: $x=0$, $y = -3(0) + 1 = 1$

$x=4$, $y = -3(4) + 1 = -11$

Range: $-11 < f(x) < 1$

FIND RANGE OF FOLLOWING FUNCTIONS:

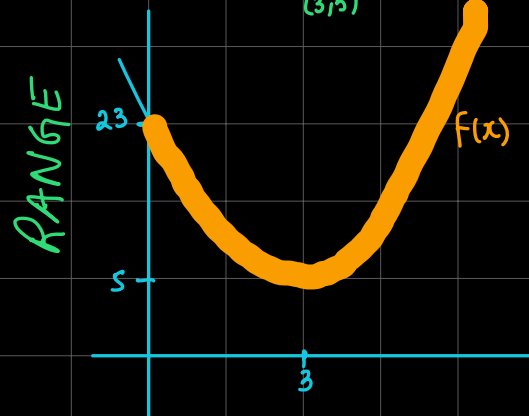
(a) $f(x) = 2(x-3)^2 + 5$



Range:

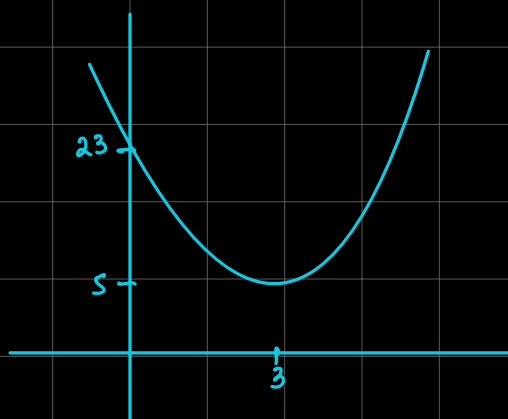
(b) $f(x) = 2(x-3)^2 + 5$ $x > 0$

Turning point: $\begin{matrix} x-3=0 \\ x=3 \end{matrix}$, $y=5$
(3,5)



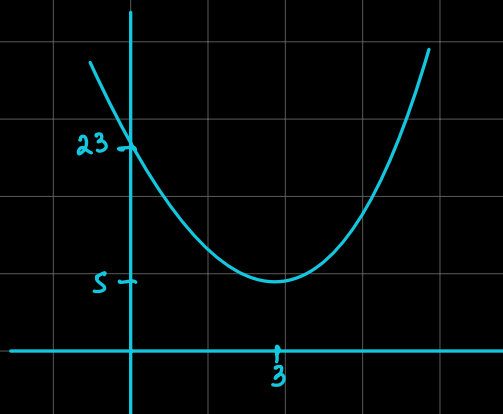
Range: $f(x) \geq 5$

(c) $f(x) = 2(x-3)^2 + 5 \quad x \geq 1$



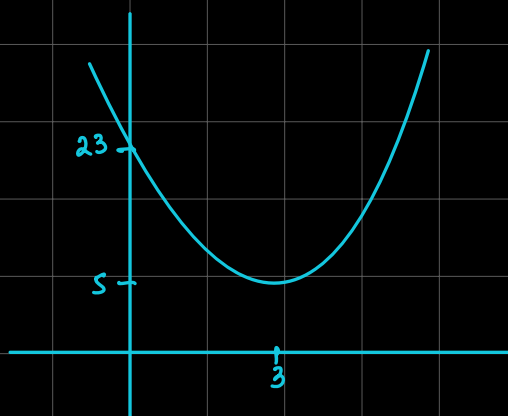
Range:

(d) $f(x) = 2(x-3)^2 + 5 \quad x < 1$



Range:

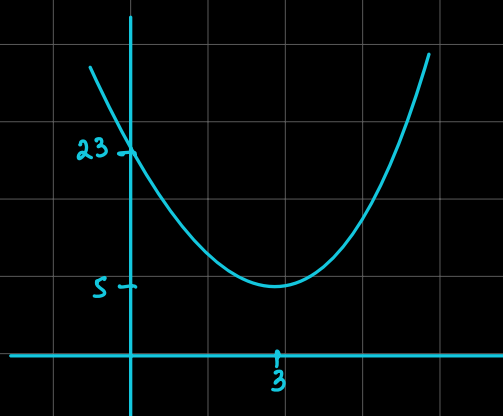
(e) $f(x) = 2(x-3)^2 + 5 \quad x \geq 5$



Range:

(f)

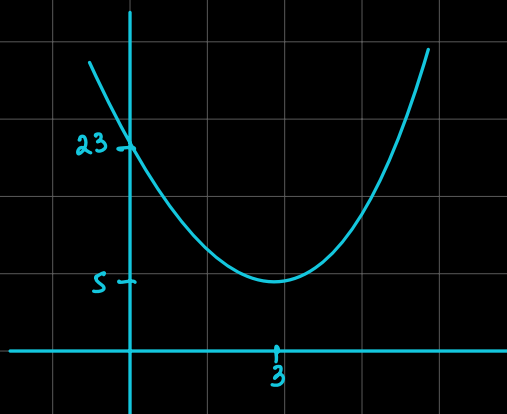
$f(x) = 2(x-3)^2 + 5 \quad 4 < x \leq 5$



Range:

(g)

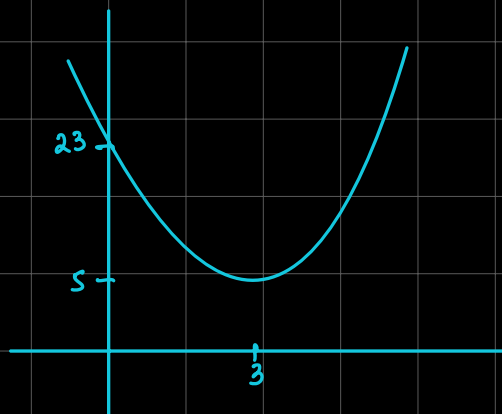
$$f(x) = 2(x-3)^2 + 5 \quad 1 < x < 4$$



Range:

(h)

$$f(x) = 2(x-3)^2 + 5 \quad x < 0$$



Range :

(i) $f(x) = 3x + 7 \quad 1 < x \leq 10$



(j) $f(x) = 3 - x \quad 0 < x < 2$

